NONLINEAR FLUCTUATIONS IN A PLASMA WITH COULOMB INTERACTION

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The theory of thermodynamic plasma fluctuations is now fairly well developed [1-3]. However, in practice, one often comes across plasma states which are very far from being in equilibrium. Fluctuations in such nonequilibrium states have been investigated by a series of authors [4-9] in terms of a linear approximation.

It must, however, be noted that, under specific conditions, neglect of nonlinear effects may turn out to be unjustified. This relates particularly to plasma states which are close to being unstable. In this region the fluctuations of various physical quantities are very large. A similar situation occurs, for example, in the experimentally observed "critical opalescence" in plasma, i.e., the anomalously strong scattering of electromagnetic waves by an unstable plasma [10]. The dependence of the transport coefficient on ion-sound oscillations at a fairly large ratio of electron and ion temperatures [11] is another example illustrating the insufficiency of the linear approximation. Finally, nonlinear effects may be significant in a plasma with highly developed turbulence.

All this points to the necessity of expressing the various correlation functions characteristic of fluctuation processes in terms of higher correlation functions. In doing so, it is natural to confine oneself, for a start, to the first approximation in order of nonlinearity.

The present paper solves this problem for plasma with Coulomb interaction.

1. As is well known [12], the state of a two-component plasma with Coulomb interaction can be described by the phase density

$$N_a(\mathbf{r}, \mathbf{p}, t) = \sum_i \delta(\mathbf{r} - \mathbf{r}_{ia}(t))\, \delta(\mathbf{p} - \mathbf{p}_{ia}(t))$$

satisfying the equation

$$\frac{\partial N_a}{\partial t} + \mathbf{v}\frac{\partial N_a}{\partial \mathbf{r}} - \sum_b \int d\mathbf{r}' d\mathbf{p}' \frac{\partial U_{ab}(|\mathbf{r}-\mathbf{r}'|, \lambda)}{\partial \mathbf{r}} N_b(\mathbf{r}', \mathbf{p}', t) \frac{\partial N_a}{\partial \mathbf{p}} = 0.$$

Here

$$U_{ab}(r, \lambda) = \frac{e_a e_b}{r}(1 - e^{-\lambda r}), \qquad \lambda > 0 \qquad (1.1)$$

is the modified Coulomb interaction, going over into Coulomb interaction for $\lambda \to \infty$. The introduction of interaction (1.1) is a generally accepted formal device, used in order to exclude the strictly energetic divergences which arise in the description of a system of particles with Coulomb interaction [13]. The Fourier transform of (1.1) differs from that of the Coulomb interaction by the factor $\lambda^2/(k^2 + \lambda^2)$, which ensures the convergence of integrals in k-space, and the significance of the above-mentioned procedure lies in the fact that the passage to the $\lambda \to \infty$ has already been realized in the end results. However, one can see that the operator $\lim (\lambda^2/k^2 + \lambda^2)$ for $\lambda \to \infty$ is equivalent to replacing the improper integrals in k-space associated with Coulomb interaction with integrals in the sense of a principal value.

This latter procedure is obviously more convenient, since it allows us to dispense with the additional factor.

We represent the phase density in the form of two components

$$N_a = \langle N_a \rangle + \delta N_a$$

the first of which represents the phase density average taken over the ensemble, while the second describes the fluctuations about the mean. By definition, $\langle \delta N_a \rangle = 0$.

Everywhere in what follows we shall consider that the plasma as a whole is neutral and that its states are those of quasi-equilibrium. The latter condition tacitly assumes that space-time variations of the mean phase density values are slow in comparison with the corresponding fluctuation scales.

It can be shown that δN_a satisfies the equations

$$\frac{\partial \delta N_a}{\partial t} + \mathbf{v}\frac{\partial \delta N_a}{\partial \mathbf{r}} - \frac{\partial \langle N_a \rangle}{\partial \mathbf{p}} \sum_b \int d\mathbf{r}' d\mathbf{p}' \frac{\partial U_{ab}(|\mathbf{r}-\mathbf{r}'|)}{\partial \mathbf{r}} \delta N_b(\mathbf{r}', \mathbf{p}', t) = \sum_b \int d\mathbf{r}' d\mathbf{p}' \frac{\partial U_{ab}}{\partial \mathbf{r}} \frac{\partial}{\partial \mathbf{p}} (\delta Q_{ab} - \langle \delta Q_{ab} \rangle)$$

$$\delta Q_{ab} = \delta N_a(\mathbf{r}, \mathbf{p}, t)\, \delta N_b(\mathbf{r}', \mathbf{p}', t). \qquad (1.2)$$

Finally, one can establish the following relations [12], which will be required later on:

$$\langle N_a \rangle = n_a f_a, \qquad (1.3)$$

$$\langle \delta N_a(\mathbf{r}, \mathbf{p}, t)\, \delta N_b(\mathbf{r}', \mathbf{p}', t) \rangle = \delta_{ab} \delta(\mathbf{r} - \mathbf{r}')\, \delta(\mathbf{p} - \mathbf{p}')\, n_b f_b + n_a n_b g_{ab}(\mathbf{r}, \mathbf{r}', \mathbf{p}, \mathbf{p}'), \qquad (1.4)$$

$$\langle \delta N_a(\mathbf{r}, \mathbf{p}, t)\, \delta N_b(\mathbf{r}', \mathbf{p}', t)\, \delta N_c(\mathbf{r}'', \mathbf{p}'', t) \rangle = \delta_{ab}\delta_{ac}\delta(\mathbf{r} - \mathbf{r}')\, \delta(\mathbf{r} - \mathbf{r}'')\, \delta(\mathbf{p} - \mathbf{p}')\, \delta(\mathbf{p} - \mathbf{p}'')\, n_c f_c + \delta_{ab}\delta(\mathbf{r} - \mathbf{r}')\, \delta(\mathbf{p} - \mathbf{p}')\, n_a n_c g_{ac} + \delta_{ac}\delta(\mathbf{r} - \mathbf{r}'')\, \delta(\mathbf{p} - \mathbf{p}'')\, n_c n_b g_{bc} + \delta_{bc}\delta(\mathbf{r}' - \mathbf{r}'')\, \delta(\mathbf{p}' - \mathbf{p}'')\, n_a n_b g_{ab} + n_a n_b n_c d_{abc}, \qquad (1.5)$$

$$g_{ab} = f_{ab} - f_a f_b, \quad d_{abc} = f_{abc} - g_{ab} f_c - g_{bc} f_a - g_{ac} f_b - f_a f_b f_c.$$

Here n_a is the number of particles of type a in unit volume, and g_{ab} and d_{abc} are double and triple correlation functions respectively.

2. Fluctuation processes in a plasma are given by the double correlation function

$$\langle \delta N_a(\mathbf{r}, \mathbf{p}, t)\, \delta N_b(\mathbf{r}', \mathbf{p}', t)\rangle. \tag{2.1}$$

When this is calculated in the framework of the linear theory, the triple simultaneous correlation function is neglected, as a result of which the right-hand side of the system of equations (1.2) is also neglected. The system then turns out to be self-consistent, which ensures the solution of the problem. In the first approximation in order of nonlinearity the correlation function (1.5) is preserved, and correspondingly the right-hand side of (1.2) is also retained. As a result, it becomes necessary to supplement the system of equations (1.2) for δQ_{ab}. However, leaving this question open until a later stage, we shall formally take the right-hand side in system (1.2) to be given and solve the initial problem with the initial condition

$$\delta N_a(\mathbf{r}, \mathbf{p}, t=0) = \delta N_a(\mathbf{r}, \mathbf{p}, 0).$$

To do this we apply the unilateral time Fourier transform and the coordinate Fourier transform

$$\delta N_a(\mathbf{k}, \mathbf{p}, \omega) = \int_0^\infty dt \int d\mathbf{r}\, \delta N_a(\mathbf{r}, \mathbf{p}, t)\, e^{i[(\omega+i0)t - \mathbf{kr}]}$$

$$\delta N_a(\mathbf{r}, \mathbf{p}, t) = \frac{1}{(2\pi)^4}\int d\mathbf{k}\int d\omega\, \delta N_a(\mathbf{k}, \mathbf{p}, \omega)\, e^{-i(\omega t - \mathbf{kr})}.$$

Straightforward calculations give

$$\delta N_a(\mathbf{r}, \mathbf{p}, t) = \delta N_a(\mathbf{r} - \mathbf{v}t, \mathbf{p}, 0) -$$

$$-\sum_b \frac{4\pi i e_a e_b}{(2\pi)^4} n_a \int d\mathbf{r}' d\mathbf{p}'\, \delta N_b(\mathbf{r}', \mathbf{p}', 0) \times$$

$$\times \int d\mathbf{k}\, d\omega\, \mathbf{k}\frac{\partial f_a}{\partial \mathbf{p}} \frac{e^{-i[\omega t - i\mathbf{k}(\mathbf{r}-\mathbf{r}')]}}{k^2(\omega + i0 - \mathbf{kv})(\omega + i0 - \mathbf{kv}')\,\varepsilon(\omega + i0, \mathbf{k})} +$$

$$+\ \delta n_a(\mathbf{r}, \mathbf{p}, t) \tag{2.2}$$

where

$$\varepsilon(\omega, \mathbf{k}) = 1 + \sum_a \frac{4\pi e_a^2}{k^2} n_a \int d\mathbf{p}\, \frac{1}{\omega - \mathbf{kv}}\, \mathbf{k}\frac{\partial f_a}{\partial \mathbf{p}}$$

$$\delta n_a(\mathbf{r}, \mathbf{p}, t) = \frac{1}{(2\pi)^4}\int d\mathbf{k}\, d\omega\, e^{-i(\omega t - \mathbf{kr})} \frac{1}{\omega - \mathbf{kv} + i0}\Big[iA_a(\mathbf{k}, \mathbf{p}, \omega) -$$

$$-\sum_b \frac{4\pi i e_a e_b}{k^2} n_a \mathbf{k}\frac{\partial f_a}{\partial \mathbf{p}} \int d\mathbf{p}'\, \frac{A_b(\mathbf{k}, \mathbf{p}', 0)}{(\omega + i0 - \mathbf{kv}')\,\varepsilon(\omega + i0, \mathbf{k})}\Big] \tag{2.3}$$

and $A_a(\mathbf{k}, \mathbf{p}, \omega)$ is the Fourier transform of

$$A_a(\mathbf{r}, \mathbf{p}, t) = \sum_b \int d\mathbf{r}' d\mathbf{p}'\, \frac{\partial U_{ab}}{\partial \mathbf{r}} \frac{\partial}{\partial \mathbf{p}} (\delta Q_{ab} - \langle \delta Q_{ab}\rangle). \tag{2.4}$$

Multiplying (2.2) by $\delta N_b(\mathbf{r}', \mathbf{p}', 0)$ and averaging the result over the ensemble, we find without difficulty

$$\langle \delta N_a(\mathbf{r}, \mathbf{p}, t)\, \delta N_b(\mathbf{r}', \mathbf{p}', 0)\rangle = \frac{1}{(2\pi)^4}\int d\mathbf{k}\, d\omega\, e^{i[\mathbf{k}(\mathbf{r}-\mathbf{r}') - \omega t]} \times$$

$$\times (\delta N_a(\mathbf{p})\, \delta N_b(\mathbf{p}'))_{\mathbf{k}, \omega+i0} + \langle \delta n_a(\mathbf{r}, \mathbf{p}, t)\, \delta N_b(\mathbf{r}', \mathbf{p}', 0)\rangle \tag{2.5}$$

where

$$(\delta N_a(\mathbf{p})\, \delta N_b(\mathbf{p}'))_{\mathbf{k}, \omega+i0} = \frac{i}{\omega + i0 - \mathbf{kv}}\Big[\delta_{ab}\delta(\mathbf{p} - \mathbf{p}')\, n_b f_b +$$

$$+\ n_a n_b G_{ab}(\mathbf{k}, \mathbf{p}, \mathbf{p}') -$$

$$-\frac{4\pi}{k^2} e_a e_b n_a n_b f_b \mathbf{k}\frac{\partial f_a}{\partial \mathbf{p}} \frac{1}{(\omega + i0 - \mathbf{kv})\,\varepsilon(\omega + i0, k)} -$$

$$-\sum_c \frac{4\pi}{k^2} e_a e_b n_a n_b n_c \mathbf{k}\frac{\partial f_a}{\partial \mathbf{p}} \frac{1}{\varepsilon(\omega + i0, \mathbf{k})} \int d\mathbf{p}''\, \frac{G_{cb}(\mathbf{k}, \mathbf{p}'', \mathbf{p}')}{(\omega + i0 - \mathbf{kv}'')}\Big]. \tag{2.6}$$

Here G_{ab} in the Fourier transform of the pair correlation function

$$g_{ab}(\mathbf{r}, \mathbf{p}, \mathbf{p}') = \int \frac{d\mathbf{k}}{(2\pi)^3} G_{ab}(\mathbf{k}, \mathbf{p}, \mathbf{p}')\, e^{i\mathbf{kr}}.$$

The first term in (2.5) coincides with the result of linear theory [6] (on condition that for the pair correlation function we use the "linear" pair correlation function). Both the last and the first term contain nonlinearity, since instead of G_{ab} we must now use the "nonlinear" pair correlation function.

If we turn out attention to the structure of the correction term in (2.5), it becomes obvious that the second term in (2.4) does not contribute to the double correlation function. Taking only the first term into account, we have

$$A_a(\mathbf{k}, \mathbf{p}, \omega) = \frac{4\pi i e_a}{(2\pi)^3}\frac{\partial}{\partial \mathbf{p}}\int d\mathbf{k}'\, \frac{\mathbf{k}'}{(k')^2} Q_a(\mathbf{k} - \mathbf{k}', \mathbf{k}', \mathbf{p}, \omega) \tag{2.7}$$

where

$$Q_a(\mathbf{k}, \mathbf{k}', \mathbf{p}, \omega) = \sum_b e_b \int d\mathbf{p}'\, \delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', \omega). \tag{2.8}$$

It is convenient to represent relation (2.5) in a slightly different form. Multiplying (2.5) on the left by the operator

$$\sum_a e_a \int d\mathbf{p}$$

we find without difficulty

$$\langle \delta\rho(\mathbf{r}, t)\, \delta N_b(\mathbf{r}', \mathbf{p}', 0)\rangle =$$

$$= \int \frac{d\mathbf{k}\, d\omega}{(2\pi)^4} e^{i[\mathbf{k}(\mathbf{r}-\mathbf{r}') - \omega t]} (\delta\rho\, \delta N_b(\mathbf{p}'))_{\mathbf{k}, \omega+i0, \mathbf{r}'} \tag{2.9}$$

where

$$(\delta\rho\, \delta N_b(\mathbf{p}'))_{\mathbf{k}, \omega+i0, \mathbf{r}'} =$$

$$\Phi_b(\mathbf{k}, \omega + i0, \mathbf{p}') + \Psi_b(\mathbf{k}, \omega + i0, \mathbf{p}', \mathbf{r}'), \tag{2.10}$$

$$\Phi_b(\mathbf{k}, \omega + i0, p')$$

$$= \frac{i n_b}{\varepsilon(\omega + i0, \mathbf{k})}\Big[\frac{e_b f_b}{\omega - \mathbf{kv}' + i0} + \sum_a e_a n_a \int d\mathbf{p}\, \frac{G_{ab}(\mathbf{k}, \mathbf{p}, \mathbf{p}')}{\omega - \mathbf{kv} + i0}\Big], \tag{2.11}$$

$$\Psi_b(\mathbf{k}, \omega + i0, \mathbf{p}', \mathbf{r}') = \frac{i}{\varepsilon(\omega + i0, \mathbf{k})} e^{i\mathbf{k}\mathbf{r}'} \int \frac{d\mathbf{k}'}{(2\pi)^3} \sum_a \frac{4\pi i}{(k')^2} e_a^2 \times$$
$$\times \int d\mathbf{p} \frac{1}{\omega + i0 - \mathbf{k}\mathbf{v}} \left(\mathbf{k}' \frac{\partial}{\partial \mathbf{p}}\right) \times$$
$$\times \langle Q_a(\mathbf{k} - \mathbf{k}', \mathbf{k}', \mathbf{p}, \omega)\, \delta N_b(\mathbf{r}', \mathbf{p}', 0)\rangle \qquad (2.12)$$

We shall see later that the dependence of Ψ_b on $\mathbf{r}'$ is a spurious one. With the help of relations (2.10) - (2.12) it is not hard to represent (2.5) in the form

$$\langle \delta N_a(\mathbf{r}, \mathbf{p}, t)\, \delta N_b(\mathbf{r}', \mathbf{p}', 0)\rangle = \int \frac{d\mathbf{k} d\omega}{(2\pi)^4} e^{i[\mathbf{k}(\mathbf{r}-\mathbf{r}') - \omega t]} \times$$
$$\times \Big[\chi_{ab}(\mathbf{k}, \omega + i0, \mathbf{p}, \mathbf{p}', \mathbf{r}') -$$
$$- \frac{4\pi}{k^2} e_a n_a \mathbf{k} \frac{\partial f_a}{\partial \mathbf{p}} \frac{(\delta\rho\, \delta N_b(\mathbf{p}'))_{\mathbf{k}, \omega + i0, \mathbf{r}'}}{\omega - \mathbf{k}\mathbf{v} + i0}\Big] \qquad (2.13)$$

where

$$\chi_{ab}(\mathbf{k}, \omega + i0, \mathbf{p}, \mathbf{p}', \mathbf{r}') = \chi_{ab}^{(1)}(\mathbf{k}, \omega + i0, \mathbf{p}, \mathbf{p}') +$$
$$+ \chi_{ab}^{(2)}(\mathbf{k}, \omega + i0, \mathbf{p}, \mathbf{p}', \mathbf{r}'), \qquad (2.14)$$

$$\chi_{ab}^{(1)}(\mathbf{k}, \omega + i0, \mathbf{p}, \mathbf{p}') = \frac{i}{\omega + i0 - \mathbf{k}\mathbf{v}} [\delta_{ab}\delta(\mathbf{p} - \mathbf{p}') n_b f_b +$$
$$+ n_a n_b G_{ab}(\mathbf{k}, \mathbf{p}, \mathbf{p}')], \qquad (2.15)$$

$$\chi_{ab}^{(2)}(\mathbf{k}, \omega + i0, \mathbf{p}, \mathbf{p}', \mathbf{r}') = \frac{i}{\omega + i0 - \mathbf{k}\mathbf{v}} e^{i\mathbf{k}\mathbf{r}'} \frac{4\pi i e_a}{(2\pi)^3} \int \frac{d\mathbf{k}'}{(k')^2} \times$$
$$\times \mathbf{k}' \frac{\partial}{\partial \mathbf{p}} \langle Q_a(\mathbf{k} - \mathbf{k}', \mathbf{k}', \mathbf{p}, \omega)\, \delta N_b(\mathbf{r}', \mathbf{p}', 0)\rangle. \qquad (2.16)$$

It should be noted that the form of the relation between the correlation function $\langle \delta N \delta N\rangle$ and $\langle \delta\rho\delta N\rangle$ is of the same form as in the linear theory [6].

3. Relations (2.9) and (2.13) are still only a formal solution of the problem posed, since the function δQ_{ab} is as yet unspecified. Thus it is necessary to construct a system of equations determining δQ_{ab}. Multiplying the equation for δN_a by δN_b and the equation for δN_b by δN_a and adding the results, we find

$$\frac{\partial \delta Q_{ab}}{\partial t} + \left(\mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \mathbf{v}'\frac{\partial}{\partial \mathbf{r}'}\right)\delta Q_{ab} -$$
$$- \sum_c \int d\mathbf{r}'' d\mathbf{p}'' \Big[n_a \frac{\partial U_{ac}}{\partial \mathbf{r}} \frac{\partial f_a}{\partial \mathbf{p}} \delta Q_{bc} + n_b \frac{\partial U_{bc}}{\partial \mathbf{r}'} \frac{\partial f_b}{\partial \mathbf{p}'} \delta Q_{ac}\Big] =$$
$$= \sum_c \int d\mathbf{r}'' d\mathbf{p}'' \Big[\frac{\partial U_{ac}}{\partial \mathbf{r}} \frac{\partial}{\partial \mathbf{p}} (\delta R_{abc} - \langle \delta Q_{ac}\rangle \delta N_b) +$$
$$+ \frac{\partial U_{bc}}{\partial \mathbf{r}'} \frac{\partial}{\partial \mathbf{p}'} (\delta R_{abc} - \langle \delta Q_{bc}\rangle \delta N_a)\Big] \qquad (3.1)$$

where

$$\delta R_{abc} = \delta N_a \delta N_b \delta N_c .$$

System (3.1) allows one to express δQ_{ab} by means of a pair distribution function, the fluctuation of phase density, and a triple simultaneous correlation function. However, if we limit ourselves to taking into account the first corrections in order of nonlinearity, then the right-hand side of (3.1) can be neglected, which automatically closes the problem.

We shall seek the solution of the initial problem for system (3.1) without right-hand side by the Fourier method. Thus,

$$\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', \omega) =$$
$$= \int_0^\infty dt \int d\mathbf{r}\, d\mathbf{r}' \delta Q_{ab}(\mathbf{r}, \mathbf{r}', \mathbf{p}, \mathbf{p}', t)\, e^{i[(\omega + i0)t - \mathbf{k}\mathbf{r} - \mathbf{k}'\mathbf{r}']},$$

$$\delta Q_{ab}(\mathbf{r}, \mathbf{r}', \mathbf{p}, \mathbf{p}', t) =$$
$$= \frac{1}{(2\pi)^7} \int d\mathbf{k}\, d\mathbf{k}' \int d\omega \delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', \omega)\, e^{-i[\omega t - \mathbf{k}\mathbf{r} - \mathbf{k}'\mathbf{r}']},$$

as a result of which we arrive at the equations

$$\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', \omega) = \frac{i\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0)}{\omega + i0 - \mathbf{k}\mathbf{v} - \mathbf{k}'\mathbf{v}'} -$$
$$- \sum_c \frac{4\pi e_a e_c}{k^2} n_a \mathbf{k} \frac{\partial f_a}{\partial \mathbf{p}} \frac{1}{\omega + i0 - \mathbf{k}\mathbf{v} - \mathbf{k}'\mathbf{v}'} \times$$
$$\times \int d\mathbf{p}'' \delta Q_{bc}(\mathbf{k}, \mathbf{k}', \mathbf{p}', \mathbf{p}'', \omega) -$$
$$- \sum_c \frac{4\pi e_b e_c}{k^2} n_b \mathbf{k}' \frac{\partial f_b}{\partial \mathbf{p}'} \frac{1}{\omega + i0 - \mathbf{k}\mathbf{v} - \mathbf{k}'\mathbf{v}'} \times$$
$$\times \int d\mathbf{p}'' \delta Q_{ac}(\mathbf{k}, \mathbf{k}', \mathbf{p}', \mathbf{p}'', \omega)$$
$$\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0) = \delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', t = 0). \qquad (3.2)$$

We note that δQ_{ab} is symmetric with respect to the substitution of a, k, p respectively by b, k′, p′ and vice versa. In solving (3 2) we shall follow the method outlined in [14, 15]. We introduce

$$Q_a(\mathbf{k}, \mathbf{k}', \mathbf{p}, \omega) = \sum_b e_b \int d\mathbf{p}' \delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', \omega).$$

Multiplying (3.2) by e_b summing over b and integrating over p′, we find

$$\varepsilon(\omega - \mathbf{k}\mathbf{v} + i0, \mathbf{k}')\, Q_a(\mathbf{k}, \mathbf{k}', \mathbf{p}, \omega) =$$
$$= \sum_b e_b \int d\mathbf{p}' \frac{i\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0)}{\omega + i0 - \mathbf{k}\mathbf{v} - \mathbf{k}'\mathbf{v}'} -$$
$$- \frac{8\pi^2 i e_a}{k^2} n_a \mathbf{k} \frac{\partial f_a}{\partial \mathbf{p}} M(\mathbf{k}', \mathbf{k}, \omega - \mathbf{k}\mathbf{v} + i0, \omega), \qquad (3.3)$$

where

$$M(\mathbf{k}, \mathbf{k}', \omega, \omega') = \frac{1}{2\pi i} \sum_a e_a \int d\mathbf{p} \frac{Q_a(\mathbf{k}, \mathbf{k}', \mathbf{p}, \omega')}{\omega - \mathbf{k}\mathbf{v}}.$$

Now multiplying (3.3) by $e_a\delta(\omega' - \omega + \mathbf{k}\mathbf{v})$, summing over a and integrating over p, we obtain

$$\varepsilon(\omega' + i0, \mathbf{k}')\, [M(\mathbf{k}, \mathbf{k}', \omega - \omega' - i0, \omega) -$$
$$- M(\mathbf{k}, \mathbf{k}', \omega - \omega' + i0, \omega)] = \qquad (3.4)$$

$$= \sum_a e_a \int d\mathbf{p} \sum_b e_b \int d\mathbf{p}' \frac{i\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0)}{\omega' - \mathbf{k}'\mathbf{v}' + i0} \delta(\omega - \omega' - \mathbf{k}\mathbf{v}) -$$

$$- M(\mathbf{k}', \mathbf{k}, \omega' + i0, \omega) \times \qquad (3.4)$$

$$\times [\varepsilon(\omega - \omega' - i0, \mathbf{k}) - \varepsilon(\omega - \omega' + i0, \mathbf{k})]. \text{(cont'd)}$$

Making the substitutions k′ and $\omega' - \omega' \to \omega'$ in Eq. (3.4), and subtracting the equation thus obtained from (3.4) itself, we have

$$\varepsilon(\omega' + i0, \mathbf{k}') M(\mathbf{k}, \mathbf{k}', \omega - \omega' - i0, \omega) +$$

$$+ \varepsilon(\omega - \omega' - i0, \mathbf{k}) \times M(\mathbf{k}', \mathbf{k}, \omega' + i0, \omega) -$$

$$- \frac{1}{2\pi i} \sum_a e_a \int d\mathbf{p}' \sum_b \frac{i\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0) e_b \, d\mathbf{p}}{(\omega' - \mathbf{k}'\mathbf{v}' + i0)(\omega - \omega' - \mathbf{k}\mathbf{v} - i0)} =$$

$$= \varepsilon(\omega' - i0, \mathbf{k}') M(\mathbf{k}, \mathbf{k}', \omega - \omega' + i0, \omega) +$$

$$+ \varepsilon(\omega - \omega' + i0, \mathbf{k}) \times M(\mathbf{k}', \mathbf{k}, \omega' - i0, \omega) -$$

$$- \frac{1}{2\pi i} \sum_a e_a \int d\mathbf{p} \sum_b e_b \int \frac{i\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0) \, d\mathbf{p}'}{(\omega' - \mathbf{k}'\mathbf{v}' - i0)(\omega - \omega' - \mathbf{k}\mathbf{v} + i0)}. \qquad (3.5)$$

Considering (3.5) as the bounding relation for a certain function of the complex variable ω', which vanishes for $|\omega'| \to \infty$, we conclude, by a well-known theorem from the theory of analytic functions, that

$$\varepsilon(\omega', \mathbf{k}') M(\mathbf{k}, \mathbf{k}', \omega - \omega', \omega) +$$

$$+ \varepsilon(\omega - \omega', \mathbf{k}) M(\mathbf{k}', \mathbf{k}, \omega', \omega) =$$

$$= \frac{1}{2\pi i} \sum_a e_a \int d\mathbf{p} \sum_b e_b \int d\mathbf{p}' \frac{i\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0)}{(\omega' - \mathbf{k}'\mathbf{v}')(\omega - \omega' - \mathbf{k}\mathbf{v})}. \qquad (3.6)$$

Relation (3.6) allows one to eliminate the quantity $M(\mathbf{k}, \mathbf{k}', \omega - \omega', \omega)$, from (3.4), as a result of which we find

$$\frac{M(\mathbf{k}', \mathbf{k}, \omega' - i0, \omega)}{\varepsilon(\omega' - i0, \mathbf{k}')} - \frac{M(\mathbf{k}', \mathbf{k}, \omega' + i0\omega)}{\varepsilon(\omega' + i0, \mathbf{k}')} =$$

$$= \sum_a e_a \int d\mathbf{p} \sum_b e_b \int d\mathbf{p}' \frac{1}{2\pi i} \frac{1}{\omega - \omega' - \mathbf{k}\mathbf{v} + i0} \times$$

$$\times i \frac{\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0)}{\varepsilon(\omega - \omega' + i0, \mathbf{k})} \times$$

$$\times \left[\frac{1}{\varepsilon(\omega' - i0, \mathbf{k}')(\omega' - \mathbf{k}'\mathbf{v}' - i0)} - \frac{1}{\varepsilon(\omega' + i0, \mathbf{k}')(\omega' - \mathbf{k}'\mathbf{v}' + i0)} \right].$$

Whence we obtain, from the Sokhotskii-Plemel formulas,

$$\frac{M(\mathbf{k}', \mathbf{k}, \omega', \omega)}{\varepsilon(\omega', \mathbf{k}')} = \frac{1}{(2\pi i)^2} \int \frac{du}{\omega' - u} \sum_a e_a \int d\mathbf{p} \sum_b e_b \int d\mathbf{p}' \times$$

$$\times \frac{i\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0)}{\varepsilon(\omega - u + i0, \mathbf{k})(\omega - u - \mathbf{k}\mathbf{v} + i0)} \times$$

$$\times \left[\frac{1}{(u - \mathbf{k}'\mathbf{v}' - i0)\, \varepsilon(u - i0, \mathbf{k}')} - \frac{1}{(u - \mathbf{k}'\mathbf{v}' + i0)\, \varepsilon(u + i0, \mathbf{k}')} \right].$$

Setting (3.7) in (3.3), we have

$$Q_a(\mathbf{k}, \mathbf{k}', \mathbf{p}, \omega) = \sum_b e_b \times$$

$$\times \int d\mathbf{p}' \frac{i\delta Q_{ab}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', 0)}{\varepsilon(\omega - \mathbf{k}\mathbf{v} + i0, \mathbf{k}')(\omega - \mathbf{k}\mathbf{v} - \mathbf{k}'\mathbf{v}' + i0)} +$$

$$+ \sum_b e_b \int d\mathbf{p}' \sum_c e_c \int d\mathbf{p}'' \frac{4\pi}{k^2} e_a n_a \mathbf{k} \frac{\partial f_a}{\partial \mathbf{p}} \frac{1}{2\pi i} \int \frac{du}{\omega - \mathbf{k}\mathbf{v} - u + i0} \times$$

$$\times \frac{i\delta Q_{bc}(\mathbf{k}, \mathbf{k}', \mathbf{p}', \mathbf{p}'', 0)}{\varepsilon(\omega - u + i0, \mathbf{k})\, \varepsilon(u + i0', \mathbf{k}')(\omega - u - \mathbf{k}'\mathbf{v}' + i0)(u - \mathbf{k}'\mathbf{v}'' + i0)}.$$

Formula (3.8) also gives the explicit expression for the function entering into the required double correlation functions (2.9)-(2.11).

4. Setting (3.8) in (2.16), representing δQ_{ab} (k, k′, p, p′, 0) by the Fourier integral in coordinate space, using (1.5) and then passing once again to the k-representation, we find after calculation

$$\chi_{ab}^{(2)}(\mathbf{k}, \omega + i0, \mathbf{p}, \mathbf{p}', \mathbf{r}') =$$

$$= \chi_{ab}^{(2)}(\mathbf{k}, \omega + i0, \mathbf{p}, \mathbf{p}') = -\frac{4\pi i}{(2\pi)^3} \int \frac{d\mathbf{k}'}{(k')^2} \times$$

$$\times \left[\frac{e_a}{\omega - \mathbf{k}\mathbf{v} + i0} \mathbf{k}' \frac{\partial}{\partial \mathbf{p}} \sum_c e_c \times \right.$$

$$\times \int d\mathbf{p}_1 \frac{N_{acb}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}_1, \mathbf{p}')}{(\omega - \Delta\mathbf{k}\mathbf{v} - \mathbf{k}'\mathbf{v}_1 + i0)\, \varepsilon(\omega - \Delta\mathbf{k}\mathbf{v} + i0, \mathbf{k}')} +$$

$$+ \frac{4\pi}{(\Delta\mathbf{k})^2} \frac{1}{2\pi i} \int du \frac{\mathbf{k}'\eta_a(\omega + i0, \mathbf{k}, \omega - u + i0, \Delta\mathbf{k}, \mathbf{v})}{\varepsilon(\omega - u + i0, \Delta\mathbf{k})\, \varepsilon(u + i0, \mathbf{k}')} \times$$

$$\times \sum_c e_c \int d\mathbf{p}_1 \sum_d e_d \int d\mathbf{p}_2 \frac{N_{cdb}(\mathbf{k}, \mathbf{k}', \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}')}{(\omega - u - \Delta\mathbf{k}\mathbf{v}_2 + i0)(u - \mathbf{k}'\mathbf{v}_1 + i0)} \qquad (4.1)$$

where $\Delta\mathbf{k} = \mathbf{k} - \mathbf{k}'$ and

$$\eta(\omega, \mathbf{k}, \omega', \mathbf{k}', \mathbf{v}) = e_a^2 n_a \frac{1}{\omega - \mathbf{k}\mathbf{v}} \frac{\partial}{\partial \mathbf{p}} \frac{1}{\omega' - \mathbf{k}'\mathbf{v}} \left(\mathbf{k}' \frac{\partial f_a}{\partial \mathbf{p}} \right) \qquad (4.2)$$

$$N_{abc}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}', \mathbf{p}'') =$$

$$= \delta_{ab}\delta_{ac}\delta(\mathbf{p} - \mathbf{p}')\,\delta(\mathbf{p} - \mathbf{p}'')\, n_c f_c + \delta_{ab}\delta(\mathbf{p} - \mathbf{p}') \times$$

$$\times n_a n_c G_{ac}(\mathbf{k}, \mathbf{p}, \mathbf{p}'') + \delta_{ac}\delta(\mathbf{p} - \mathbf{p}'')\, n_c n_b G_{cb}(-\mathbf{k}', \mathbf{p}'', \mathbf{p}') +$$

$$+ \delta_{bc}\delta(\mathbf{p}' - \mathbf{p}'') \times n_b n_a G_{ab}(\mathbf{k}' - \mathbf{k}, \mathbf{p}, \mathbf{p}') +$$

$$+ n_a n_b n_c D_{abc}(\mathbf{k}, -\mathbf{k}', -\mathbf{k} + \mathbf{k}', \mathbf{p}, \mathbf{p}', \mathbf{p}'') \qquad (4.3)$$

$$d_{abc} = \frac{1}{(2\pi)^6} \int d\mathbf{k}\, d\mathbf{k}' e^{i[(\mathbf{k}+\mathbf{k}')\mathbf{r} - \mathbf{k}'\mathbf{r}' - \mathbf{k}\mathbf{r}'']} \times$$

$$\times D_{abc}(\mathbf{k}, \mathbf{k}', -\mathbf{k} - \mathbf{k}', \mathbf{p}, \mathbf{p}', \mathbf{p}''). \qquad (4.4)$$

It still remains to calculate Ψ_d. To do this we employ the obvious relationship

$$\Psi_b = \frac{1}{\varepsilon(\omega + i0, \mathbf{k})} \sum_a e_a \int d\mathbf{p}\, \chi_{ab}^{(2)}$$

as a result of which we obtain

$$\Psi_b = -\frac{4\pi i}{(2\pi)^3 \varepsilon(\omega + i0, \mathbf{k})} \times$$

$$\times \int \frac{d\mathbf{k}'}{(k')^2} \Big\{ \sum_a e_a^2 \int d\mathbf{p} \sum_c e_c \int d\mathbf{p}_1 \frac{1}{\omega - \mathbf{kv} + i0} \times$$

$$\times \mathbf{k} \frac{\partial}{\partial \mathbf{p}} \frac{N_{acb}(\mathbf{k}, \mathbf{k}', \mathbf{p}, \mathbf{p}_1, \mathbf{p}')}{(\omega - \Delta\mathbf{kv} - \mathbf{k}'\mathbf{v}_1 + i0)\, \varepsilon(\omega - \Delta\mathbf{kv} + i0, \mathbf{k}')} +$$

$$+ \frac{1}{2\pi i} \int \frac{du\, \mathbf{k}' \eta(\omega + i0, \mathbf{k}, \omega - u + i0, \Delta\mathbf{k})}{\varepsilon(\omega - u + i0, \Delta\mathbf{k})\, \varepsilon(u + i0, \mathbf{k}')\, \Delta} \frac{4\pi}{k^2} \times$$

$$\times \sum_c e_c \int d\mathbf{p}_1 \sum_d e_d \int d\mathbf{p}_2 \frac{N_{cdb}(\mathbf{k}, \mathbf{k}', \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}')}{(\omega - u - \Delta\mathbf{kv}_2 + i0)(u - \mathbf{k}'\mathbf{v}_1 + i0)} \Big\}$$

where

$$\eta(\omega, \mathbf{k}, \omega', \mathbf{k}') = \sum_a e_a \int d\mathbf{p} \eta_a(\omega, \mathbf{k}, \omega', \mathbf{k}', \mathbf{v})$$

is the function introduced in [15].

The relations (2.9)-(2.11) and (4.5) determine, in the first approximation in order of nonlinearity, the double correlation function of charge density with phase density through the first distribution function, and pair and triple correlation functions. Moreover, in order to preserve the order of smallness, the correlation functions substituted in (2.11) should correspond to the first approximation in order of nonlinearity, so that, just as in relation (4.5), we must confine ourselves to linear correlation functions.

The analogous relations (2.13)-(2.15) and (4.1) together with the Fourier transform (2.10), determined above, represent the double phase density correlator in terms of correlation functions, and are thus, in fact, the solution of the problem posed.

REFERENCES

1. M. A. Leontovich and S. M. O. Rytov, "On the differential law of intensity of electrical fluctuations, and the influence of the skin effect," ZhETF, vol. 23, no. 3, 1952.

2. L. D. Landau and E. M. Lifshitz, Electrodynamics of Continua [in Russian], Gostekhizdat, 1957.

3. V. P. Silin and A. A. Rukhadze, Electromagnetic Properties of Plasma and of Plasma-Like Media [in Russian], Atomizdat, 1961.

4. I. Hubbard, "The friction and diffusion coefficients of the Fokker-Planck equation in a plasma," Proc. Roy. Soc. A, vol. 260, p. 114, 1961.

5. V. P. Silin, "Towards a theory of electromagnetic plasma fluctuations," ZhETF vol. 41, no. 3, 1961.

6. Yu. L. Klimontovich and V. P. Silin, "Towards a theory of fluctuations of particle distributions in plasma," ZhETF, vol. 42, no. 1, 1962.

7. N. Rostoker, "Fluctuations in plasma," Nucl. Fusion, vol. 1, 1961.

8. A. I. Akhiezer, I. A. Akhiezer, and A. G. Sitenko, "Towards a theory of fluctuation distributions in plasma," ZhETF, vol. 41, no. 2, 1961.

9. L. S. Bogdankevich, A. A. Rukhadze, and V. P. Silin, "On the fluctuations of the electromagnetic field in a nonequilibrium plasma," Radiofizika, vol. 5, no. 6, 1962.

10. S. Ichimaru, D. Pines, and N. Rostoker, "Observation of critical fluctuations associated with plasma - wave instabilities," Phys. Rev. Letters, vol. 8, no. 6, 1962.

11. V. P. Silin and L. M. Gorbunov, "On the kinetics of a nonisothermal plasma," AN SSSR, vol. 145, no. 6, 1962.

12. Yu. L. Klimontovich, "Second quantization in phase space," AN SSSR, vol. 96, no. 1, 1954.

13. N. N. Bogolyubov, Dynamical Theory Problems in Statistical Physics [in Russian], Gostekhizdat, 1946.

14. V. P. Silin, "Triple correlations in a plasma, and the collision integral for the pair correlation function," Izv. VUZ. Fizika [Soviet Physics Journal], no. 1, 1965.

15. V. P. Silin, "Towards a kinetic theory of plasma wave interaction," PMTF, no. 1, p. 32, 1964.

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